

1. A) State the definition of the natural logarithm function, $\ln(x)$.

B) Use the definition of the logarithm and the First Fundamental Theorem of Calculus to prove the exponent property: For all $x > 0$, and for all rational numbers, r : $\ln(x^r) = r \ln(x)$

2. Find $\frac{d}{dx} [x^{\sin(x)}]$

3. Use implicit differentiation to establish that $\frac{d}{dx} \arccos(x) = -\frac{1}{\sqrt{1-x^2}}$

4. Consider the function: $y = \frac{1}{x^2+1}$ on $[0, \sqrt{3}]$

A) Find the area under the curve.

B) Find the volume of the solid formed if the region bounded by the curve and the x-axis on $[0, \sqrt{3}]$ is rotated about the y-axis.

5. Solve the indicated integrals. Show your steps!

A) $\int_e^{e^2} \frac{1}{x\sqrt{\ln(x)}} dx$

B) $\int \sin^2(5x) dx$

C) $\int \frac{\text{Arctan}(x)}{x^2} dx$

D) $\int_1^{\infty} \frac{\ln(x) dx}{x^2}$

6. Use L'Hospital's rule to evaluate the limit: $\lim_{x \rightarrow 0^+} (1 + \sin(3x))^{1/x}$

7. Recall that the Maclaurin series for $f(x) = \frac{1}{1+x}$ is $\sum_{n=0}^{\infty} (-1)^n x^n$

A) Find the radius of convergence, and check the convergence at the endpoints.

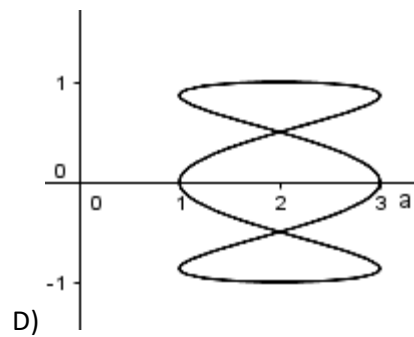
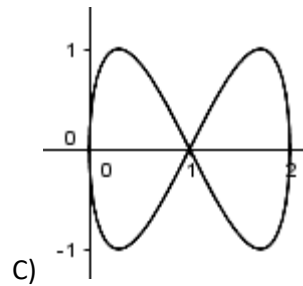
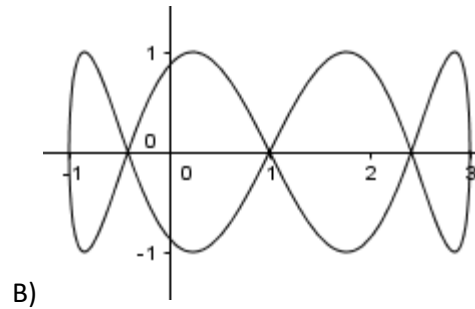
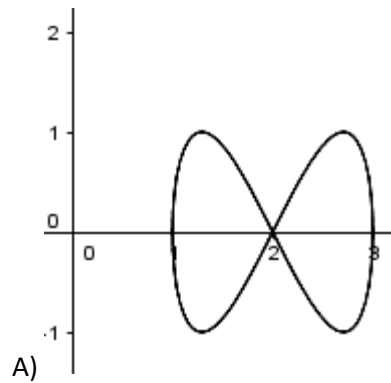
B) Use the above fact to find the Maclaurin series for $\ln(1+x)$.

8. Decide whether the series: $\sum_{n=2}^{\infty} \frac{(-1)^n \sqrt{n}}{\sqrt{n^2-n}}$ diverges, converges conditionally, or converges absolutely. Show the steps carefully!

9. Find the 3rd degree Taylor polynomial centered at $x = 8$ for the function: $f(x) = x^{2/3}$.

10.

Match the parametric graphs (A – D) with the equations (I – IV)



I

$$\left. \begin{array}{l} x = \cos(2t) + 1 \\ y = \sin(4t) \end{array} \right\} 0 \leq t \leq 6.28$$

II

$$\left. \begin{array}{l} x = 2 \cos(t) + 1 \\ y = \sin(4t) \end{array} \right\} 0 \leq t \leq 6.28$$

III

$$\left. \begin{array}{l} x = 2 \cos^2(t) + 1 \\ y = \sin(4t) \end{array} \right\} 0 \leq t \leq 6.28$$

IV

$$\left. \begin{array}{l} x = 2 \cos^2(3t) + 1 \\ y = \sin(2t) \end{array} \right\} 0 \leq t \leq 6.28$$